BASIC INFORMATION
Name: TC004F
Description: Groundwater Pollution Temporal Reconstruction
Type: Function Estimation
Unknowns: 100
Data Points: 100

FORWARD PROBLEM
Problem Type: Linear
Mathematical Model:

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2} - v\frac{\partial C}{\partial x}; \tag{3.8a}$$

$$C(0,t) = C_{in}(t); \tag{3.8b}$$

$$\lim_{x\to\infty} C(x,t) = 0. \tag{3.8c}$$

Analytical solution at time T given by

$$C(x,T) = \int_0^T C_{in}(t)\,K(x,T-t)dt, \tag{3.9}$$

with kernel function given by

$$K(x,\tau) = \frac{x}{2\sqrt{\pi D\tau^3}}\exp\left\{-\frac{[x-v\tau]^2}{4D\tau}\right\} \tag{3.10}$$

Numerical Solution: Midpoint rule integration;
Independent Parameters: $t \in (0;t_f]$, $t_f = 300$; $n_t = 100$; $t_i = i\Delta t$; $\Delta t = 3$; $x \in (0;L]$, $L = 300$; $n_x = 100$; $x_i = i\Delta x$; $\Delta x = 3$; $D = 1$; $v = 1$.
Function to be estimated: $C_i = C_{in}(t_i)$, $i = 1,\ldots,n_t$, so that $\mathbf{x} = [C_1 \quad C_2 \quad \ldots \quad C_N]^T$
Exact Function: Based on Skaggs & Kabala (1994), *Recovering the release history of a groundwater contaminant, Water Resources Research, v.30, No. 1, 71–79.*

$$C_{in}(t) = \exp\left[-\frac{(t-130)^2}{2(5^2)}\right] + 0.3\exp\left[-\frac{(t-150)^2}{2(10^2)}\right] + 0.5\exp\left[-\frac{(t-190)^2}{2(7^2)}\right] \tag{3.11}$$

EXPERIMENTAL DATA
Type: *Synthetic;*
Dataset size: $n_t = 100$*;*
Noise: *Zero mean Gaussian with std* $\sigma_y = 10^{-2}$*;*
Download of Synthetic Data: *"tc004fy.dat" file with* (t_i, y_i^{exact}, y_i)*. Remaining columns contains benchmark results and residuals, respectively.*

REGULARIZATION PARAMETER SELECTION
Selection Method(s): *GCV;*
Selected Parameter: $\lambda = 6.792627E-02$*;*
Plot: *Cf. Fig. 3.16 (GCV).*

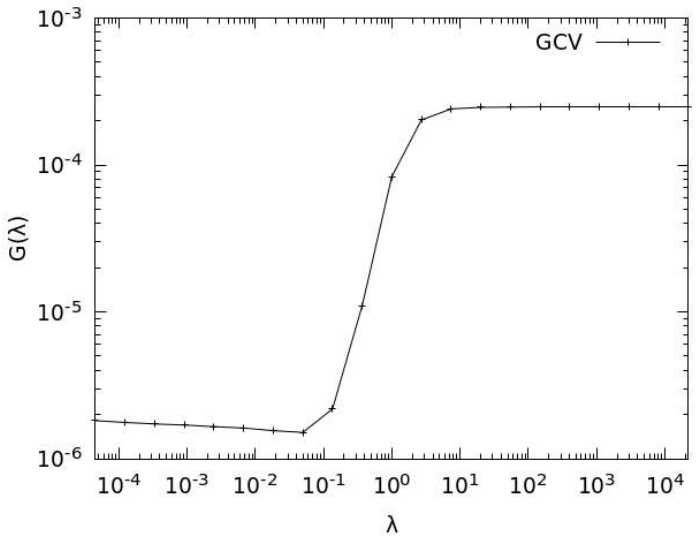

Figure 3.16: GCV curve for problem TC003F.

INVERSE PROBLEM
Solution Method: *Direct Solution via Singular Value Decomposition (SVD);*
Regularization: *0-th order Tikhonov with* $\lambda = 6.792627E-02$*;*
Plots: *Exact vs. Estimated values (cf. Fig. 3.17) and Mapping reconstruction (cf. Fig 3.18).*

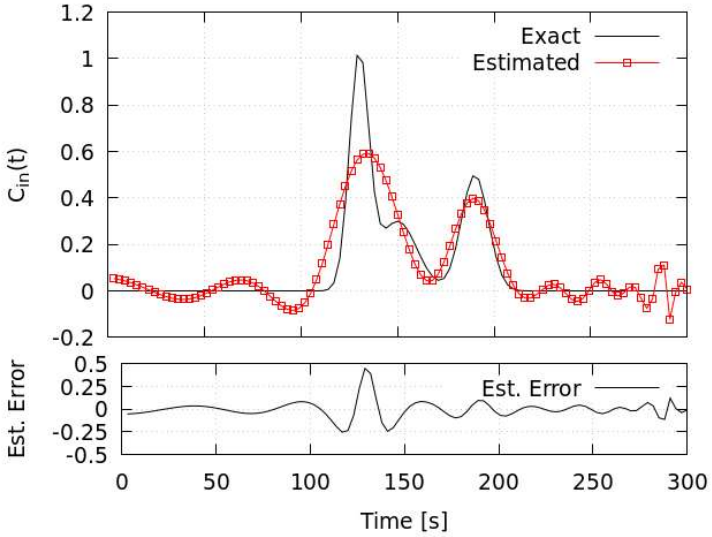

Figure 3.17: Exact and estimated profiles for $C_{in}(t)$, along with estimation error.

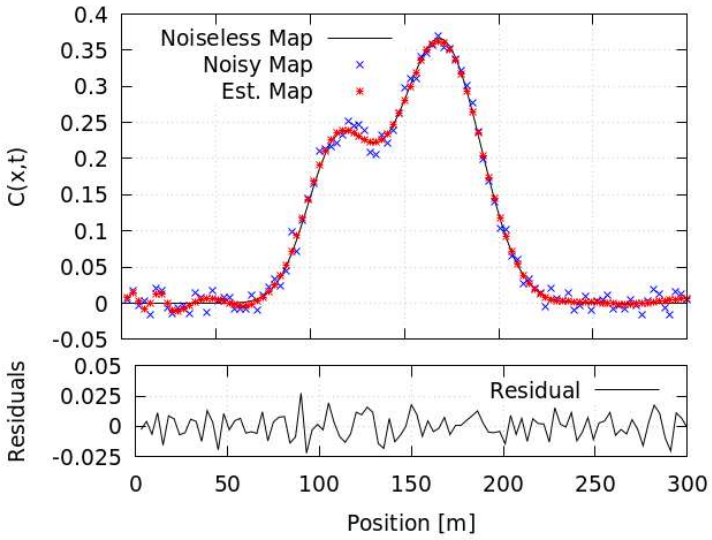

Figure 3.18: Noiseless and Synthetic Measurements, Mapped Solution and Residuals.