BASIC INFORMATION
Name: TC004P
Description: 1D Heat Conduction in Semi-Infinite Media
Type: Parameter Estimation
Unknowns: 2
Data Points: 200

FORWARD PROBLEM
Problem Type: Linear
Mathematical Model:

$$T(x,t) = T_0 \text{erf}\left(\frac{x}{\sqrt{4\alpha t}}\right), \quad x,t > 0. \tag{2.9}$$

Independent Parameters: $t \in (0;t_f]$, $t_f = 600$ s; $t_i = i\Delta t$; $\Delta t = 3$ s;
Parameters to be Estimated: T_0 and α, so that $\mathbf{x} = [x_1 \quad x_2]^T = [T_0 \quad \alpha]^T$
Exact Values:

$$T_0 = 25\ ^\circ\text{C}; \tag{2.10a}$$
$$\alpha = 10^{-5}\ \text{m}^2/\text{s}. \tag{2.10b}$$

EXPERIMENTAL DATA
Type: Temperature measurements at $x = 0.1$ m throughout $t \in [0,t_f]$ – Synthetic;
Dataset size: $N = 200$;
Noise: Zero mean Gaussian with std $\sigma_y = 1\ ^\circ$C;
Download of Synthetic Data: "TC004P_data.txt" file with (t_i, y_i^{exact}, y_i).
Plot: Cf. Fig. 2.13

INVERSE PROBLEM
Solution Method: Levenberg-Marquardt method;
Plots: Mapping reconstruction (cf. Fig 2.14).
Sensitivity coefficients: Cf. Figs. 2.15 and 2.16 for the sensitivity coefficients at the initial guess and Figs. 2.17 and 2.18 for the coefficients at the estimated values.
Estimated parameters: Cf. Tab. 2.3.

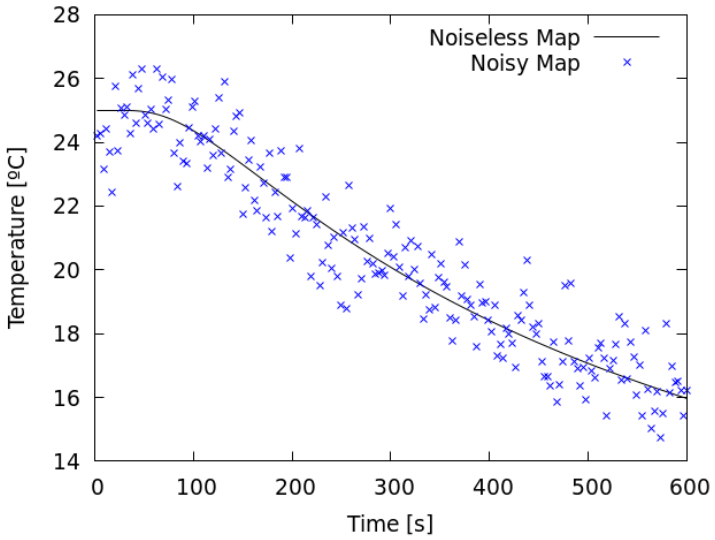

Figure 2.13: Synthetic and Noiseless Measurements.

Table 2.3: Initial, exact and estimated values of the paramters.

	Exact	Initial	Estimated
T_0	$0.250000E+02$	$0.100000E+02$	$0.247692E+02$
α	$0.100000E-04$	$0.100000E-03$	$0.960761E-05$

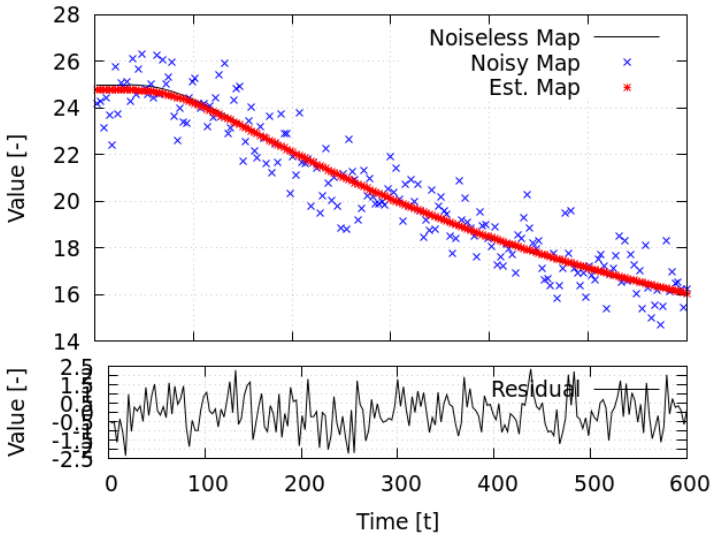

Figure 2.14: Synthetic Measurements, Mapped Solution and Residuals.

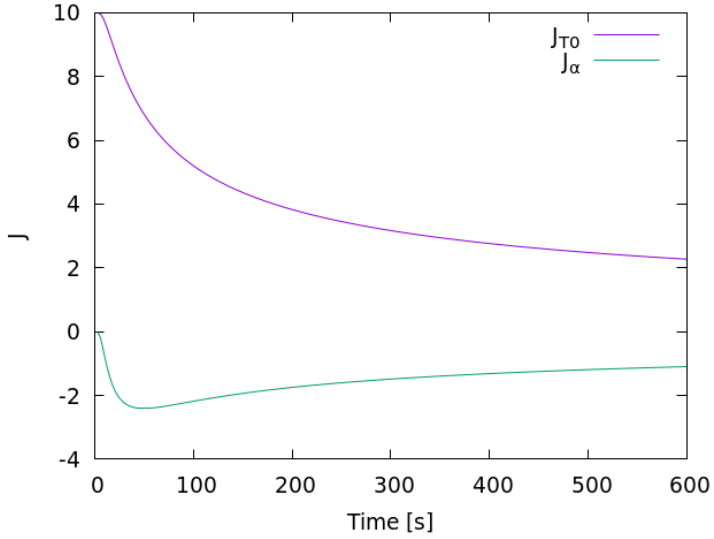

Figure 2.15: Sensitivity coefficients at initial guess.

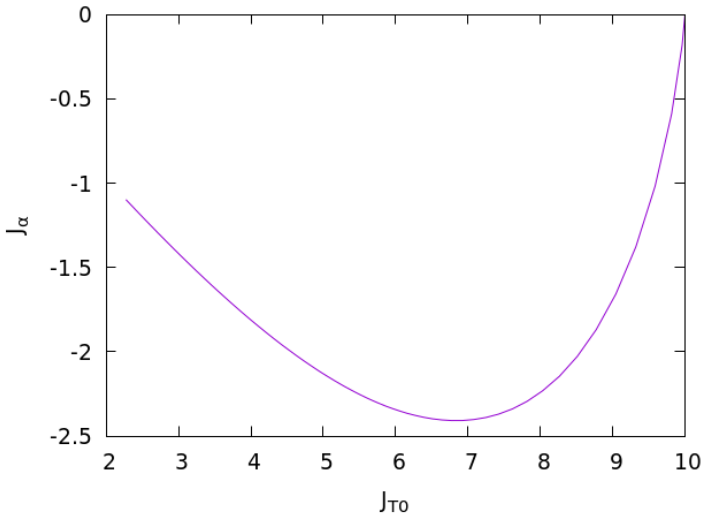

Figure 2.16: Phase diagram of the sensitivity coefficients at initial guess.

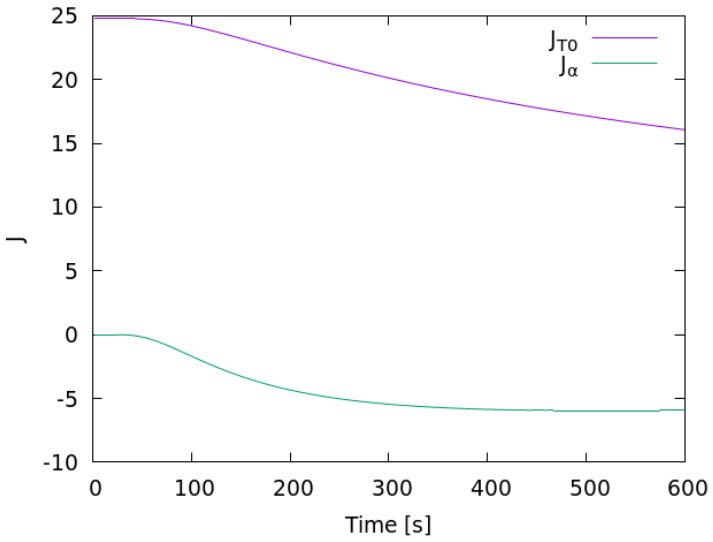

Figure 2.17: Sensitivity coefficients at estimated values.

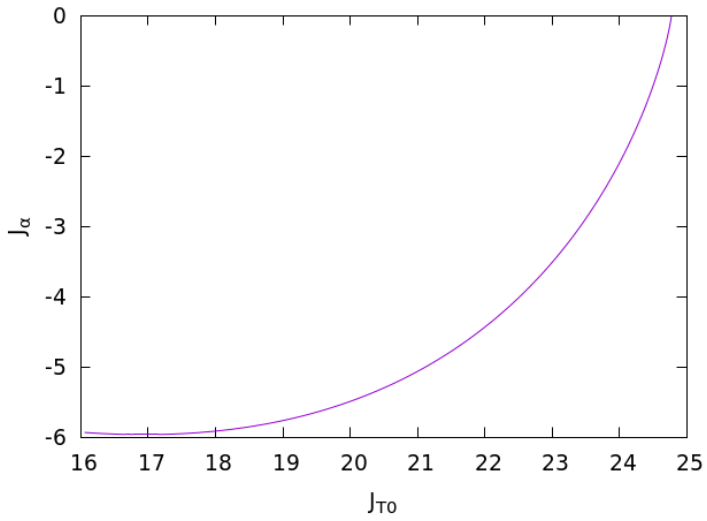

Figure 2.18: Phase diagram of the sensitivity coefficients at the estimated values.