BASIC INFORMATION
Name: TC003P
Description: Lumped Capacity Heat Conduction
Type: Parameter Estimation
Unknowns: 2
Data Points: 100

FORWARD PROBLEM
Problem Type: Linear
Mathematical Model:

$$\beta_1 \frac{d\eta}{dt} + \beta_0 \eta = q(t), \quad t > 0; \tag{2.5a}$$

$$\eta = \eta_0, \quad t = 0. \tag{2.5b}$$

with:

$$q(t) = q_0(2 + \cos \omega t) \tag{2.6}$$

Numerical Solution: Explicit Euler integration;
Initial Conditions:

$$\eta_0 = 1. \tag{2.7}$$

Independent Parameters: $t \in (0; t_f]$, $t_f = 600$ s; $\quad t_i = i\Delta t$; $\quad \Delta t = 6$ s;
Parameters to be Estimated: q_0 and ω, so that $\mathbf{x} = [x_1 \quad x_2]^T = [q_0 \quad \omega]^T$
Exact Values:

$$q_0 = 1.0; \tag{2.8a}$$

$$\omega = 0.1. \tag{2.8b}$$

EXPERIMENTAL DATA
Type: Values of η throughout $t \in [0, t_f]$ – Synthetic;
Dataset size: $N = 100$;
Noise: Zero mean Gaussian with std $\sigma_y = 10^{-1}$;
Download of Synthetic Data: "TC003P_data.dat" file with (t_i, y_i^{exact}, y_i).
Plot: Cf. Fig. 2.7

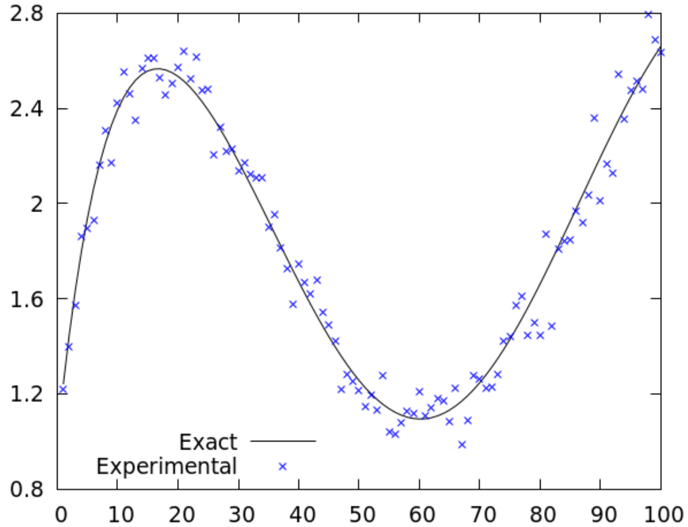

Figure 2.7: Synthetic and Noiseless Measurements.

INVERSE PROBLEM

Solution Method: Levenberg-Marquardt method;

Plots: Mapping reconstruction (cf. Fig 2.8).

Sensitivity coefficients: Cf. Figs. 2.9 and 2.10 for the sensitivity coefficients at the initial guess and Figs. 2.11 and 2.12 for the coefficients at the estimated values.

Estimated parameters: Cf. Tab. 2.2.

Table 2.2: Initial, exact and estimated values of the paramters.

	Exact	Initial	Estimated
q_0	1.0	5×10^{-2}	1.0008577556653417
ω	0.1	5×10^{-2}	0.0993499546927617

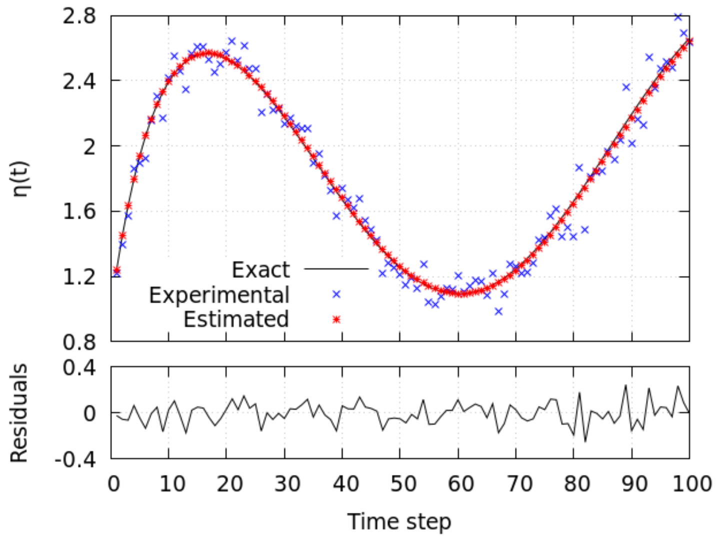

Figure 2.8: Synthetic Measurements, Mapped Solution and Residuals.

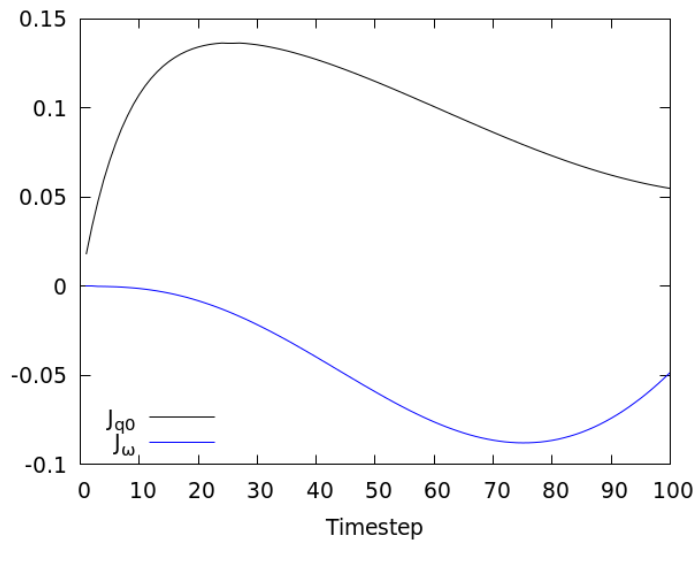

Figure 2.9: Sensitivity coefficients at initial guess.

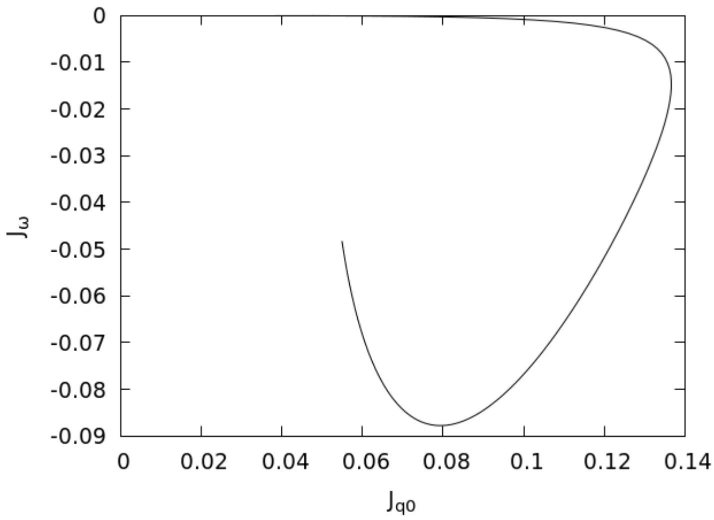

Figure 2.10: Phase diagram of the sensitivity coefficients at initial guess.

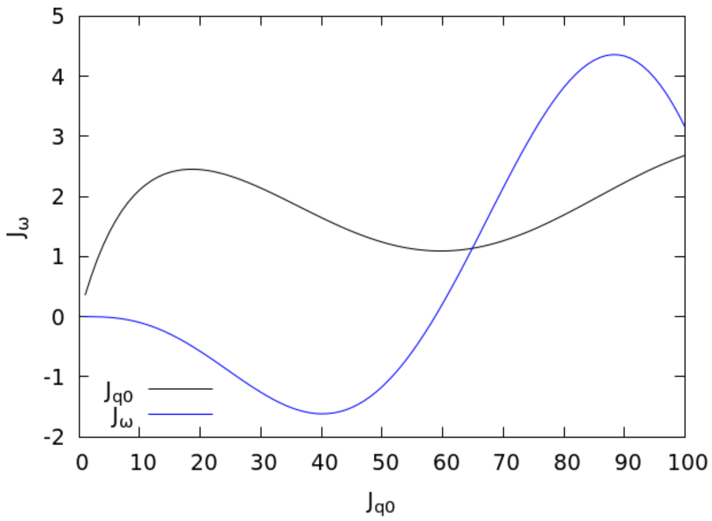

Figure 2.11: Sensitivity coefficients at estimated values.

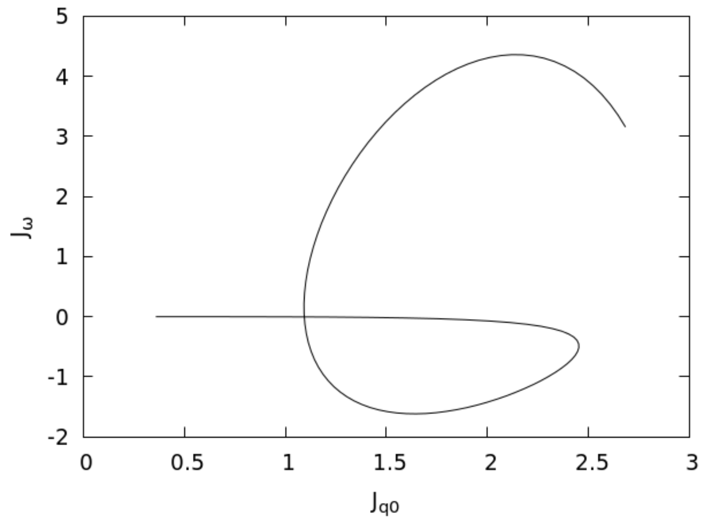

Figure 2.12: Phase diagram of the sensitivity coefficients at the estimated values.