BASIC INFORMATION
Name: TC003F
Description: Lumped Capacity Heat Conduction Model
Type: Function Estimation
Unknowns: 100
Data Points: 100

FORWARD PROBLEM
Problem Type: Linear
Mathematical Model:

$$\beta_1 \frac{d\eta}{dt} + \beta_0 \eta = q(t), \quad t > 0; \tag{3.5a}$$

$$\eta = \eta_0, \quad t = 0. \tag{3.5b}$$

Numerical Solution: Explicit Euler integration;
Initial Conditions:

$$\eta_0 = 1. \tag{3.6}$$

Independent Parameters: $t \in (0; t_f]$, $t_f = 600$ s; $n_t = 50$; $t_i = i\Delta t$; $\Delta t = 6$ s; $\beta_0 = 1$; $\beta_1 = 5$.
Parameters to be Estimated: $q_i = q(t_i)$, $i = 1, \ldots, n_t$, so that $\mathbf{x} = [q_1 \quad q_2 \quad \ldots \quad q_N]^T$
Exact Parameters:

$$q(t) = q_0(2 + \cos \omega t) \tag{3.7}$$

Plot: Cf. Fig. 3.10

EXPERIMENTAL DATA
Type: Synthetic;
Dataset size: $n_t = 50$;
Noise: Zero mean Gaussian with std $\sigma_y = 0.2$ °C;
Download of Synthetic Data: "TC003F_data.dat" file with (t_i, y_i^{exact}, y_i).
Plot: Cf. Fig. 3.11

REGULARIZATION PARAMETER SELECTION
Selection Method(s): L-curve and Morozov;
Selected Parameter: $\lambda = 10^4$;
Plot: Cf. Fig. 3.12 (L-curve) and Fig. 3.13 (Morozov).

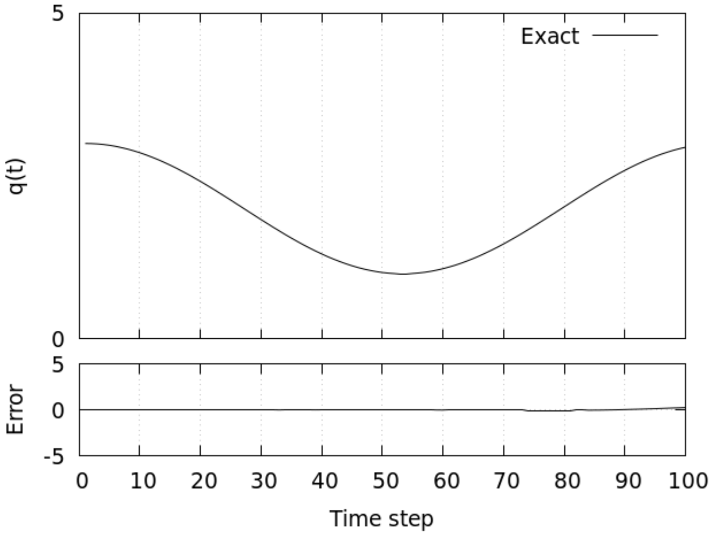

Figure 3.10: Exact heat flux $q(t)$ given in Eq. (3.7).

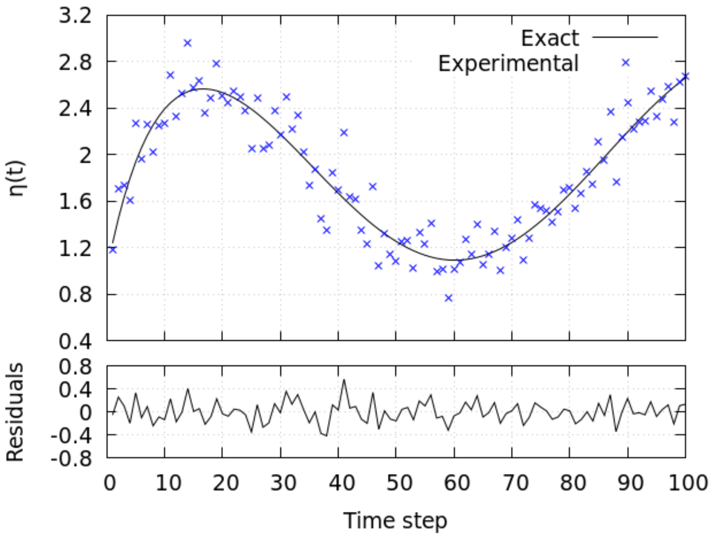

Figure 3.11: Exact and synthetic measurements.

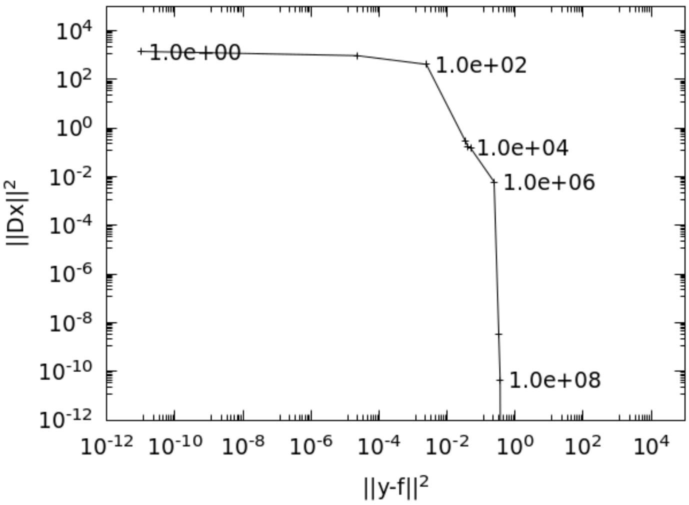

Figure 3.12: L-curve for problem TC003F.

Solution Method: Iterative Newton-Gauss solution;
Regularization: 0-th order Tikhonov with $\lambda = 10^4$;
Plots: Exact vs. Estimated values (cf. Fig. 3.14) and Mapping reconstruction (cf. Fig 3.15).

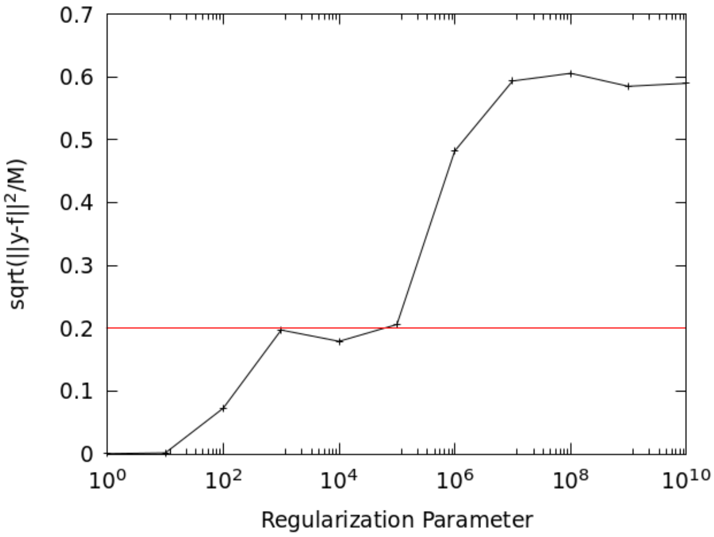

Figure 3.13: Morozov criteria for problem TC003F.

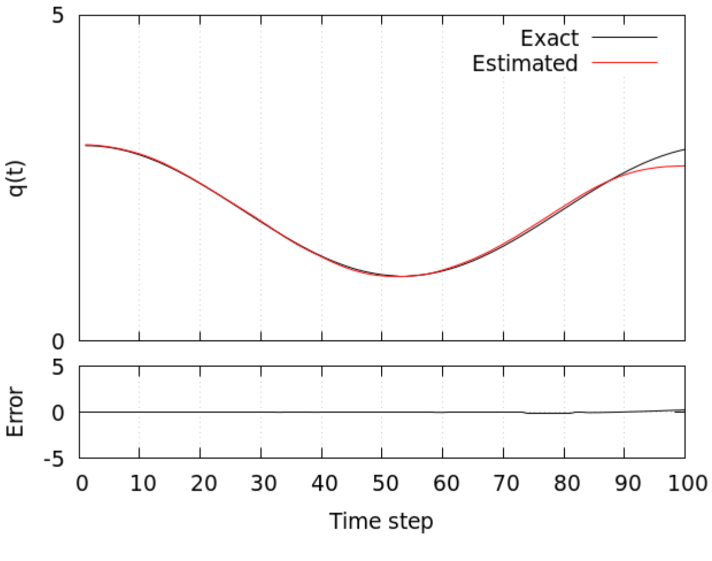

Figure 3.14: Exact and estimated profiles for $q(t)$, along with estimation error.

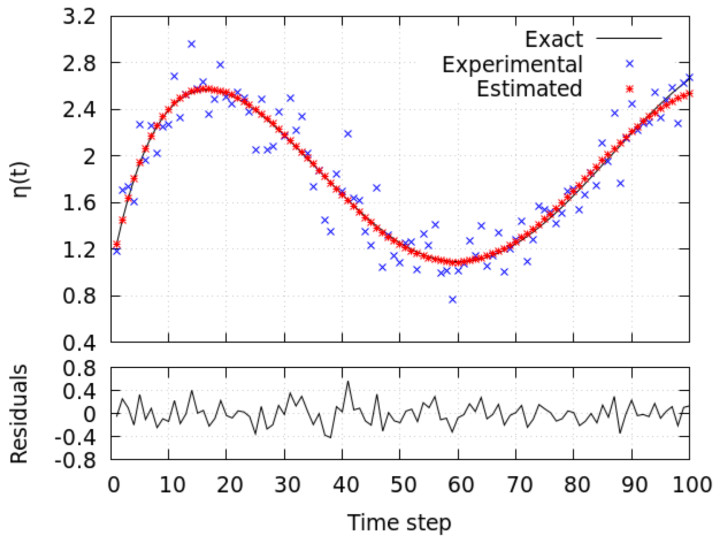

Figure 3.15: Synthetic Measurements, Mapped Solution and Residuals.