BASIC INFORMATION
Name: TC002F
Description: 1D Transient Inverse Heat Conduction
Type: Function Estimation
Unknowns: 50
Data Points: 5000

FORWARD PROBLEM
Problem Type: Linear
Mathematical Model:

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[k(x)\frac{\partial T}{\partial x}\right], \quad x \in [0,L], \quad t \in [0,t_f] \tag{3.2}$$

$$T = T_A, \quad x = 0, \quad t \in [0,t_f]; \tag{3.3a}$$
$$T = T_B, \quad x = L, \quad t \in [0,t_f]; \tag{3.3b}$$
$$T = T_0, \quad x \in [0,L], \quad t = 0. \tag{3.3c}$$

Numerical Solution: Implicit Finite Volume Method;
Independent Parameters: $L = 0.15$ m; $t_f = 600$ s; $\Delta x = 3 \times 10^{-3}$ m; $\Delta t = 6$ s; $\rho = 7830$ kg/m^3; $c_p = 434$ J/kg°C; $T_A = 150$ °C; $T_B = 100$ °C; $T_0 = 200$ °C.
Exact Parameters:

$$k(x) = \begin{cases} 100 \text{ W/m°C} & L/3 < x < 2L/3 \\ 50 \text{ W/m°C} & \text{elsewhere} \end{cases}. \tag{3.4}$$

Plot: Cf. Fig. 3.6

EXPERIMENTAL DATA
Type: Synthetic;
Dataset size: $N = 5000$;
Noise: Zero mean Gaussian with std $\sigma_y = 1$ °C;
Download of Synthetic Data: "TC002F_data.dat" file with (t_i, y_i^{exact}, y_i).

REGULARIZATION PARAMETER SELECTION
Selection Method(s): GCV;
Selected Parameter: $\lambda = 3.736 \times 10^{-1}$;
Plot: Cf. Fig. 3.7.

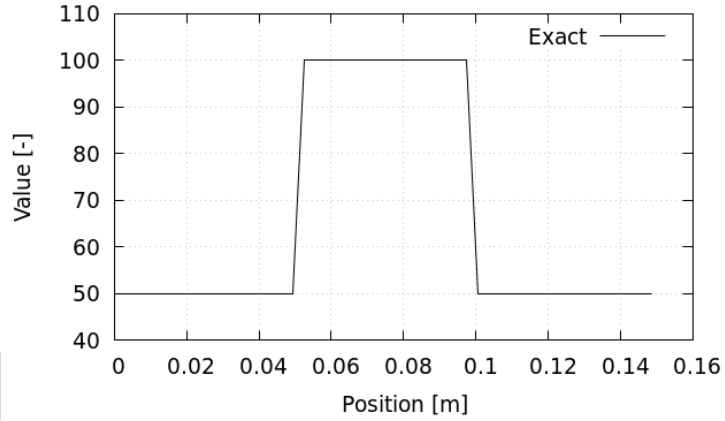

Figure 3.6: Exact thermal conductivity $k(x)$ given in Eq. (3.4).

INVERSE PROBLEM
Solution Method: Iterative Newton-Gauss solution;
Regularization: 0-th order Tikhonov with $\lambda = 10^{-2}$;
Plots: Exact vs. Estimated values (cf. Fig. 3.8) and Mapping reconstruction (cf. Fig 3.9).

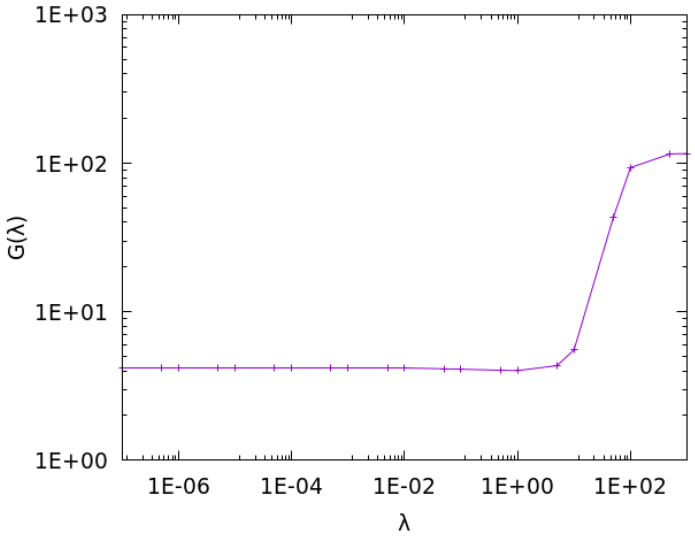

Figure 3.7: GCV curve for problem TC002F.

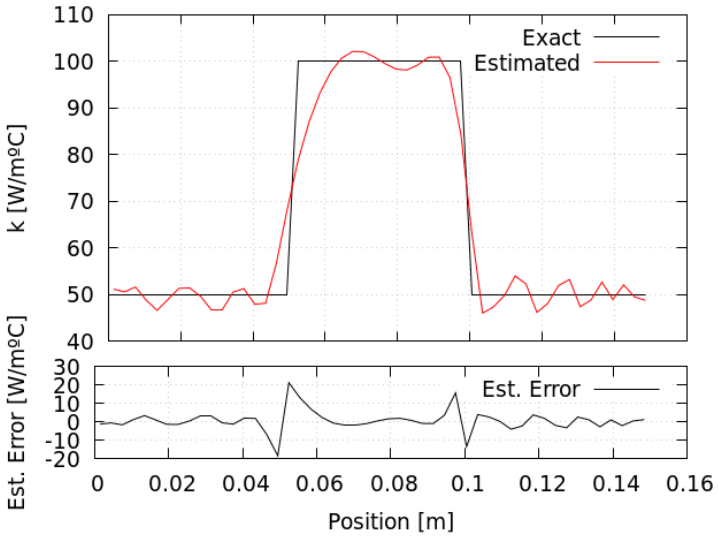

Figure 3.8: Exact and estimated profiles for $k(x)$, along with estimation error.

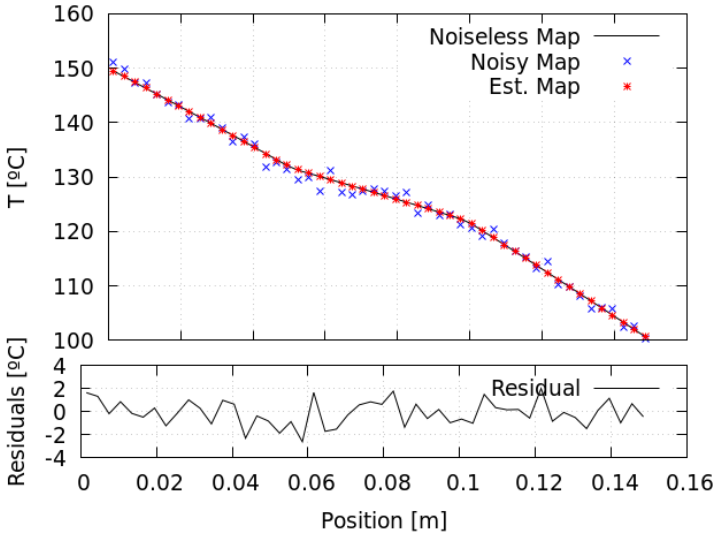

Figure 3.9: Synthetic Measurements, Mapped Solution and Residuals.