BASIC INFORMATION
Name: TC002P
Description: SIR Epidemics Model
Type: Parameter Estimation
Unknowns: 2
Data Points: 100

FORWARD PROBLEM
Problem Type: Nonlinear
Mathematical Model:

$$\frac{dS}{dt} = -\beta SI; \tag{2.1a}$$
$$\frac{dI}{dt} = \beta SI - \gamma I; \tag{2.1b}$$
$$\frac{dR}{dt} = \gamma I. \tag{2.1c}$$

Numerical Solution: 4th order Runge-Kutta integration;
Initial Conditions:

$$I_0 = 1/N_{pop}, \quad R_0 = 0, \quad S_0 = 1 - I_0 - R_0, \tag{2.2}$$

where $N_{pop} = 1000$.
Independent Parameters: $t \in (0;100]$; $\quad t_i = i\Delta t$; $\quad \Delta t = 1$;
Exact Parameters:

$$\beta = 0.8; \tag{2.3a}$$
$$\gamma = 0.6. \tag{2.3b}$$

EXPERIMENTAL DATA
Type: Total number of infected individuals $TI(t)$ up to t – Synthetic;
Expression:

$$TI(t) = \int\limits_{s=0}^{t} \beta S(s)I(s)ds. \tag{2.4}$$

Dataset size: $N = 100$;
Noise: Zero mean Gaussian with std $\sigma_y = 10^{-3}$;
Download of Synthetic Data: "TC002P_data.dat" file with (t_i, y_i^{exact}, y_i).
Plot: Cf. Fig. 2.1

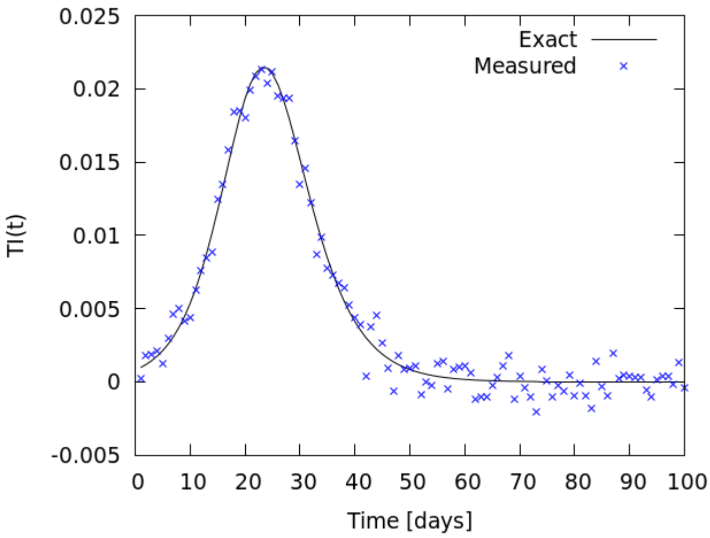

Figure 2.1: Synthetic and Noiseless Measurements.

INVERSE PROBLEM
Solution Method: Levenberg-Marquardt method;
Plots: Mapping reconstruction (cf. Fig 3.5).
Sensitivity coefficients: Cf. Figs. 2.3 and 2.4 for the sensitivity coefficients at the initial guess and Figs. 2.5 and 2.6 for the coefficients at the estimated values.
Estimated parameters: Cf. Tab. 2.1.

Table 2.1: Initial, exact and estimated values of the paramters.

	Exact	Initial	Estimated
β	0.8	0.2	0.80727195880725067
γ	0.6	0.1	0.60709305027197336

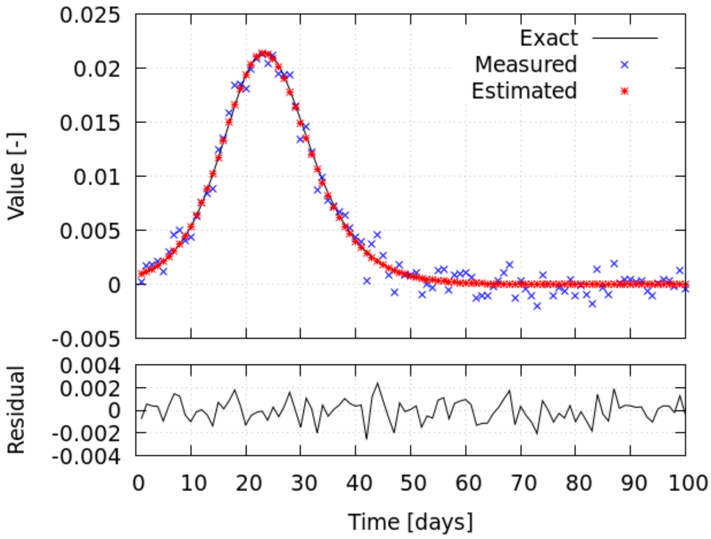

Figure 2.2: Synthetic Measurements, Mapped Solution and Residuals.

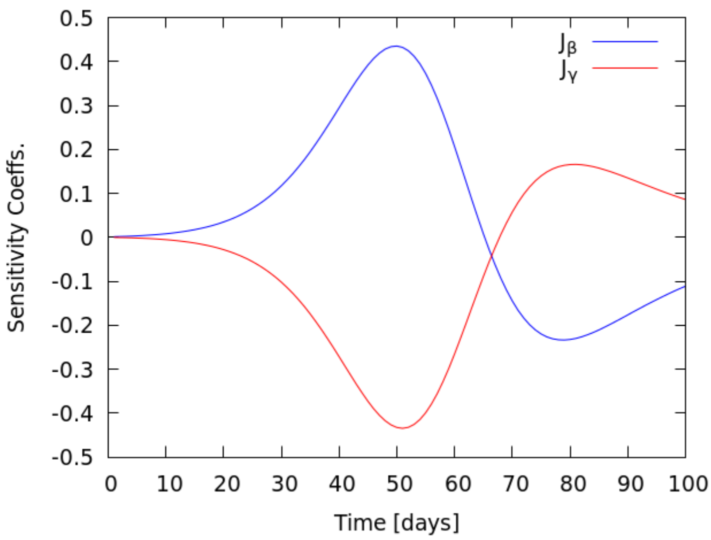

Figure 2.3: Sensitivity coefficients at initial guess.

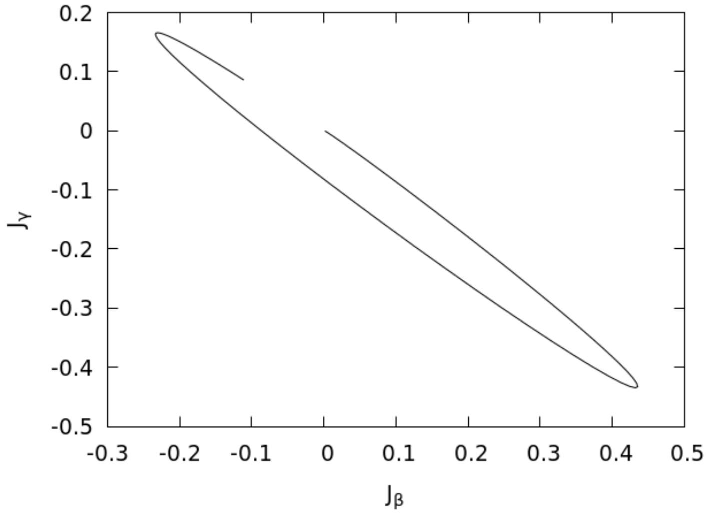

Figure 2.4: Phase diagram of the sensitivity coefficients at initial guess.

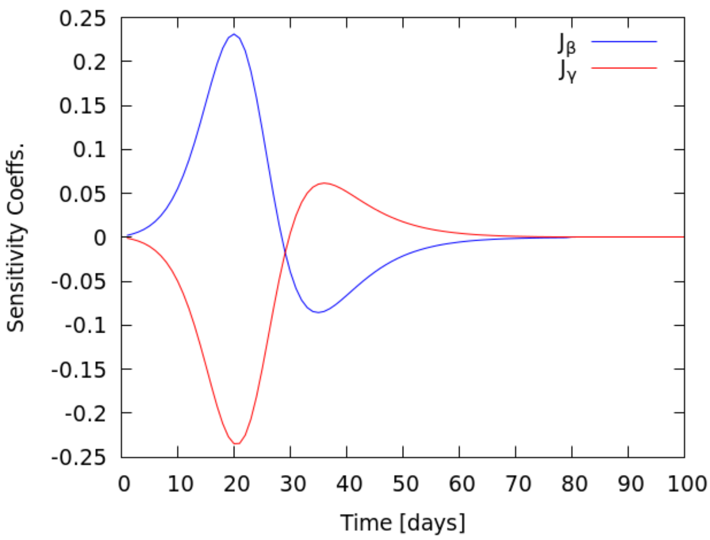

Figure 2.5: Sensitivity coefficients at estimated values.

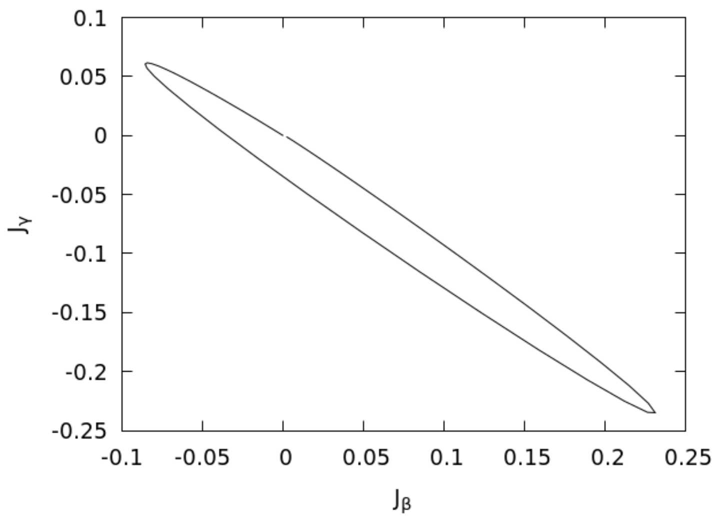

Figure 2.6: Phase diagram of the sensitivity coefficients at the estimated values.