BASIC INFORMATION
Name: TC001F
Description: 1D Fredholm Integral
Type: Function Estimation
Unknowns: 100
Data Points: 100

FORWARD PROBLEM
Problem Type: Linear
Mathematical Model:

$$y(s) = \int_0^1 K(s,t) f(t) dt.$$

Kernel type: Gaussian with $\gamma = 0.5$.

$$K(s,t) = \frac{1}{\gamma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{t-s}{\gamma}\right)^2\right] \tag{3.1}$$

Numerical Solution: Trapezoidal rule;
Independent Parameters: $s,t \in (0;1]$; $\quad t_i = i\Delta t$; $\quad \Delta t = 10^{-3}$;
Exact Parameters:

$$f(t) = \begin{cases} 1 & t_0 < t < t_1 \\ 0 & \text{elsewhere} \end{cases}, \quad \begin{matrix} t_0 = 0.4 \\ t_1 = 0.6 \end{matrix}.$$

Plot: Cf. Fig. 3.1

EXPERIMENTAL DATA
Type: Synthetic;
Dataset size: $N = 1000$;
Noise: Zero mean Gaussian with std $\sigma_y = 5 \times 10^{-2}$;
Download of Synthetic Data: "TC001F_data.dat" file with $(t_i, f_i, y_i^{exact}, y_i)$.
Plot: Cf. Fig. 3.2

REGULARIZATION PARAMETER SELECTION
Selection Method(s): L-Curve.
Plot: Cf. Fig. 3.3.

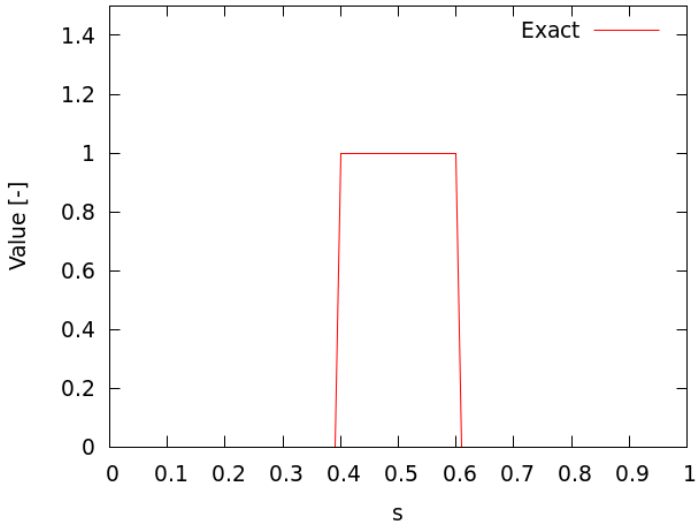

Figure 3.1: Exact profile.

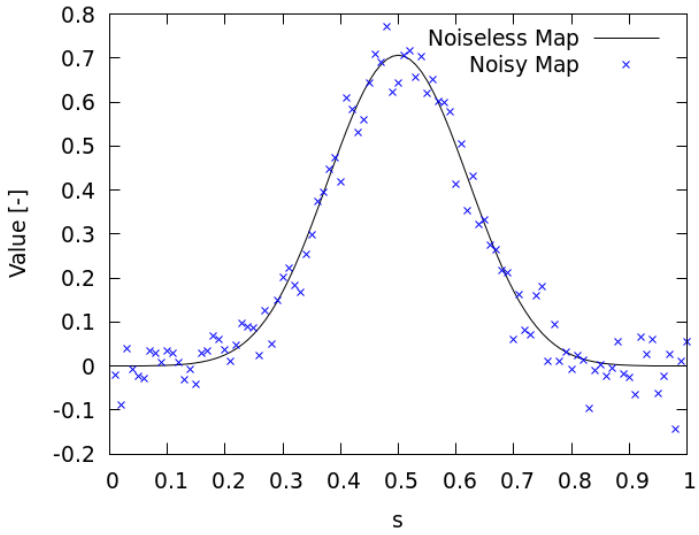

Figure 3.2: Synthetic and Noiseless Measurements.

INVERSE PROBLEM
Solution Method: Direct calculation of pseudo-inverse;

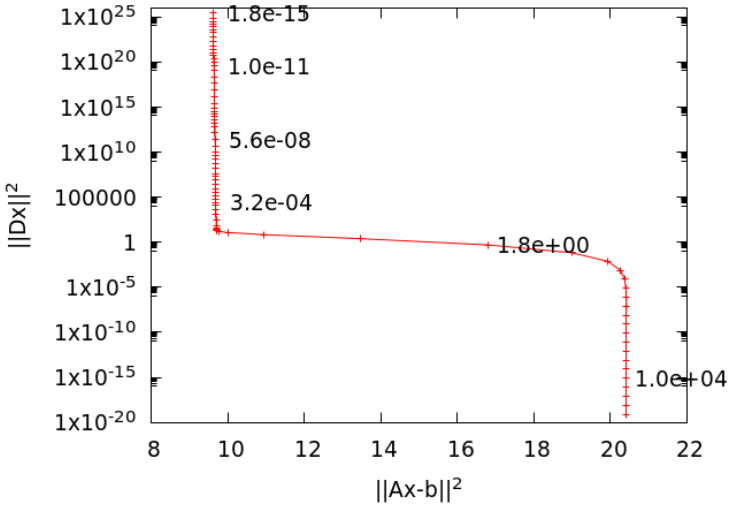

Figure 3.3: L-curve for regularization parameter selection. Optimal value is $\lambda = 10^{-2}$.

Regularization: 0-th order Tikhonov with $\lambda = 10^{-2}$;
Plots: Exact vs. Estimated values (cf. Fig. 3.4) and Mapping reconstruction (cf. Fig 3.5).

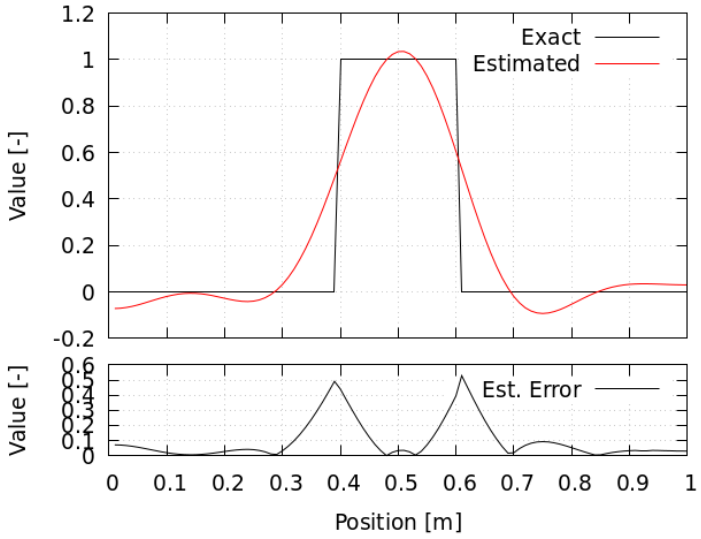

Figure 3.4: Estimates for $\lambda = 10^{-2}$.

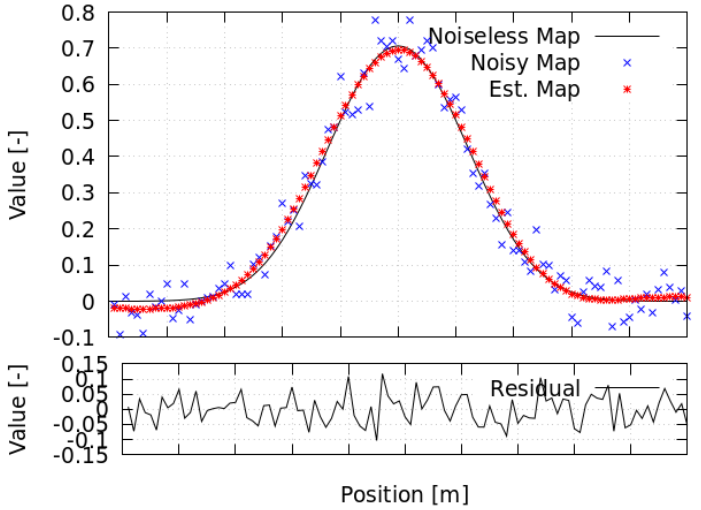

Figure 3.5: Synthetic Measurements, Mapped Solution and Residuals.